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more special equation, so that the problem awaiting solution is one of *integral* values only.

DISCUSSIONS.

The following note is a reply to that of Professor E. T. Bell (*1920*, 413), in which the use of mathematical induction in elementary teaching was criticized on logical grounds. Professor Bell's position was that the method required the postulation of a special logical principle, and was therefore unsuited to beginners.

On Proofs by Mathematical Induction.

By R. S. Hoar, South Milwaukee, Wis.

The objections to the usual method of stating mathematical induction, raised by Professor Bell in the Monthly of November, 1920, seem to call for some further discussion.

When I was a boy, the part that confused *me* was: "We first establish a theorem for $n = 1$; then we show that, if it is true for $n - 1$, it is true for n." But, if n is 1, $n - 1$ must be zero. We have not proved it for zero, and we already know that it is true for n. Of course, such an attitude is absurd, even though natural. But is it absurd to object that we never prove the proposition for $n - 1$ equals anything (but merely for n equals something), and therefore never lay the foundation for the second step?

Why not state the process as follows? "We first show that, if the proposition is true for n, it is true for $n + 1$; then we show that it is true for a particular value of n, namely $n = 1$; then, since any whole number can be reached from any preceding whole number by successively adding 1 a finite number of times, the proposition is true for n equals any whole number."

The false proof that any assemblage contains an infinity of members, cited by Prof. Bell, is not based on mathematical induction in the form objected to by him, but rather is based upon the layman's substitute for mathematical induction; namely, that if a proposition is true for $n = 1$, $n = 2$, $n = 3$, etc., repeated a "reasonable" number of times, then it must be true for any value of n.

The fact that such "proofs" are current among freshmen is probably due to the failure of their teachers to emphasize the necessity of taking the abstract general step from n to $n + 1$; just as the currency of "proofs" that two equals one, is due to the failure of teachers to introduce non-divisibility by zero at the very start, as a fundamental part of the concept of division, instead of dragging it in later as a rather lame exception, in order to defend division from the onslaughts of two-equals-one.

A real example of the non-applicability of mathematical induction would be the series set forth on page 15 of Huntington's *The Continuum* (Cambridge, Mass., 1917):

$$1_1\ 2_1\ 3_1 \cdots; \qquad 1_2\ 2_2\ 3_2 \cdots; \qquad \cdots.$$

Perhaps it would be well for teachers to exhibit both this series and the "infinite assemblage," in order that the students may realize the necessity for all three steps in mathematical induction.

REMARKS BY THE EDITOR.

Mathematical induction may be criticized with regard either to its logical basis or to its practical intelligibility. Professor Bell's contribution was of the former kind, and Mr. Hoar's of the latter. Philosophy apart, the difficulty about a proof by induction will be in making "the abstract general step from n to $n+1$." Certainly the irregular use of n and $n-1$ to which allusion has been made is a needless source of confusion, though perhaps not very serious in the long run. There are of course many to whom the very idea of the abstract general step is a hidden mystery. Such persons constitute a perennial problem in elementary teaching.

While we have the subject before us, it may be of interest to note that mathematical induction is only one of a number of processes by which theorems are extended over a wider range according to the following general plan:

It is known of a theorem T: (1) that T is true for numbers lying in a given range N_0; (2) that whenever T is true in any range N, T is also true in the range $\Omega(N)$ derived from N by means of the operations forming the set Ω. Then T is true for the numbers of any range which can be reached from N_0 by a finite number of successive applications of Ω.

Special cases are:

(*a*) N_0 consists of the number 0, and Ω consists of the operation which, applied to any set of consecutive integers, gives the same set with the next higher integer added. This is ordinary mathematical induction, proving results for all positive integers.

(*b*) The same except that it is the next lower integer that is added. This proves theorems for all negative integers.

(*c*) N_0 consists of all integers; Ω includes division by any positive integer. This proves theorems for all rational numbers.

(*d*) Cases where Ω includes the operation of adjoining to a set of numbers the limits of the set.

Some of the proofs given for the binomial and exponential theorems for general real exponents are combinations of these four types of argument.

(*e*) N_0 is the interval $(0, x_0)$; Ω consists of the operation which replaces any interval $(0, x)$ by $(0, f(x))$, where f is a positive function. This extends theorems to the interval $(0, u)$, where u is the least upper bound of $(x_0, fx_0, ffx_0, \cdots)$.

The special case of (*e*) in which $f(x) = x + k$, leading to theorems on all positive numbers, was given in detail by Y. R. Chao,[1] and illustrated with a "thin end of the wedge" fallacy. In the illustration x_0 is the measure of a dose of an intoxicant, too small to be harmful. By using a plausibly small constant k it is argued that any amount is harmless. This optimistic conclusion is reached by correctly applying the method to rather questionable assumptions.

Errors perhaps occur more often from incorrect use of (*e*) with data which are reasonable enough. For an extreme example: An athlete is known to be able to jump any height up to five feet. However high he may jump, he could always have gone a little higher (which is at least as easy to admit as the contrary). Hence he could jump any height. But in reality, $x_0 = 5$ and $(5, f(5), ff(5), \cdots)$ is a *convergent* sequence whose limit is probably less than 7.

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[1] "A note on 'Continuous Mathematical Induction,'" *Bulletin of Amer. Math. Soc.*, vol. 26, 1919, p. 17.

RECENT PUBLICATIONS.

REVIEWS.

Dynamic Symmetry, the Greek Vase. By JAY HAMBIDGE. New Haven, Yale University Press, 1920. 161 pages, small folio. Price $6.00.

A mild sensation has been caused among classicists, artists and the philosophers of æsthetics by the doctrine of "dynamic symmetry" as recently enunciated and promulgated by Mr. Jay Hambidge and his converts. Certain ambitious claims have been made which have been greeted, possibly with scepticism and repugnance by some artists and classicists, but certainly with interest and sympathy by many readers, including the Yale School of Fine Arts, and the members of various museum staffs. The exposition has been clothed in a certain amount of numerical terminology, so that many who would, perhaps, desire to approach the subject with an open mind may have been discouraged at the start and led to regard the matter as too technical for the comprehension of any one not deeply versed in the intricacies of higher mathematics.

So far as the reviewer is aware, this work has not received from professional mathematicians the attention that it deserves, particularly since any adequate criticism of the results claimed can come only from one prepared properly to appraise the mathematical background of the argument. The book under review is assuredly not intended as a reference work in geometrical analysis, but in its wider philosophical aspects, its claim for the historical and logical application of geometry to design must merit the consideration of any one who is interested in the place of mathematical discipline among the cultural developments of civilization.

Let us first cite some passages in which Mr. Hambidge enunciates his claims. "Some twenty years ago, the writer, being impressed by the incoherence of modern design and convinced that there must exist in nature some correlating principle which could give artists a control of areas, undertook a comparative study of the bases of all design, both in nature and in art. This labor resulted in the determination of two types of symmetry or proportion, one of which possessed qualities of activity, the other of passivity. For convenience, the active type was termed dynamic symmetry, the other static symmetry. It was found that the passive was the type that was employed most naturally by artists, either consciously or unconsciously; in fact no design which could be recognized as such—unless indeed it were dynamic—would be possible without the use, in some degree, of this passive or static type. . . . A study of the basis of design in art shows that this active symmetry was known to but two peoples, the Egyp-